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THE RATE OF CONSTRAINED PARTICLES DEPOSITION IN A WIDE RANGE OF SUSPENSION DENSITIES IN THE LAMINAR-TURBULENT OPERATING MODES

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Abstract. The rate of constrained fall of mineral particles in suspensions of different densities is necessary for calculating the design and operating modes of gravity concentrating equipment. During hydraulic classification and separation, a mixed, laminar-turbulent, flow regime is observed in real pulps. There are no theoretical velocity formulas for such a regime, and most of the known semi-experimental and experimental formulas have limited application. This article proposes a new method for comparing different formulas with each other in a wide range of suspension densities. The method uses an analytical calculation of the hydraulic characteristics of the medium - porosity, concentration and viscosity. What is new is that all these characteristics depend on only one indicator - the density of the suspension, which is easily determined in practice by weighing a pulp sample. In these calculations, the weighted average density of heterogeneous particles in suspension is used. A feature of the method is the approximation of the analyzed calculation formulas to the conditions of free fall in order to obtain only one control point and compare it with known experimental data. This method allows to set the limits of the application of formulas depending on the density of the suspension. The choice of a more precise formula is necessary for the design hydraulic devices for the classification and separation of mineral suspensions and finely ground composite raw materials. The application of this method for the most common formulas of Ergan and Todes-Rosenbaum is shown. It was found that the latter formula is actually not suitable for dilute suspensions with a density below 1.65 g/cm^3. A new linear equation for calculating the rate of constrained particles deposition is proposed, which provides high accuracy in a wide range of suspension densities. The resulting equation has a simple form and, together with an analytical calculation of the characteristics of the medium, forms a system of linear equations for calculating the rate of constrained particles deposition of any size and density in mineral pulps of different densities. The calculation of the velocity of constrained settling and the ascent of particles is the basis for the design of hydraulic classifiers and separators for mineral dressing. Such calculations are necessary for determining of hydraulic devices optimal modes and monitoring indicators during their operation.

Keywords: mineral suspension, density, rate of constrained deposition.

1. Introduction

The processes of hydraulic classification and separation of fine mineral suspensions (pulps) are based on the separation of dissimilar particles due to different rates of deposition. The peculiarity is that this separation takes place in a constrained environment (media). For free deposition, especially in rarefied media, the issue of determining the velocity has been studied quite well [1]. The fundamental approach is to allocate a number of regions on the Rayleigh curve depending on the Reynolds number Re. At the beginning of the Rayleigh curve, at $Re < 0.5$, the Stokes law applies. For the beginning of the transition region $0.5 < Re < 30$, a separate interpolation formula is used. For the middle of the transition region $30 < Re < 300$, the Allen formula is used. At the end of the transition region $300 < Re < 3000$ is the interpolation formula, and at $Re > 3000$ is the Newton-Rettinger formula. All these formulas, except for Stokes' law, were obtained experimentally.

Instead of the Reynolds number, P.V. Lyashenko proposed using dimensionless parameters A and B. This made possible to avoid uncertainty in determining the type of a particular formula. However, free deposition is practically not realized during the dressing of mineral pulps. Even in the simplest case of thickeners or deslimers, the particles are compacted during deposition and the process becomes constrained. The determination of the rate of constrained deposition, which is closest to the real processes of hydraulic classification and separation, presents certain difficulties.

All formulas for free fall include the viscosity and density of the suspension. These values increase during the transition to a constrained fall. However, taking into account only this fact was not enough - the velocity of a constrained fall in comparison with a free one decreases much faster. It was proposed to additionally introduce a lowering coefficient for the rate of free fall. R. Richards (1908) was the first who showed that this coefficient depends on the density, particle size and density of the suspension [2]. A significant number of such coefficients are given in the monograph by J. Happel and G. Brenner [3], they have the form of a complex function of the concentration of particles or porosity (the volume part of the liquid in the suspension).

However, the known formulas usually have limited application and include experimentally determined constants. For example, the formula of S.I. Godin (1959) is limited to a concentration of 0.3% [1], the lowering coefficient, according to P.V. Lyashenko (1948), has the form of a power function, which requires experimental determination of the degree indicator [2].

Obviously, the lowering coefficients need to be introduced separately for each of the above areas on the Rayleigh curve. Taking this into account, a number of scientists, for example, A.N. Planovsky (1967) proposed semi-empirical formulas of constrained precipitation separately for concentrated and dilute suspensions. However, most researchers operate only with the initial domain and Stokes' law. This is explained by the fact that in most models of constrained deposition, the condition of laminar flow is accepted, which occurs without separation of the flow from the particle, at low velocities with a predominance of friction forces. The semi-experimental formulas of R.T. Hancock (1937), A.M. Mitrofanov (1949), B.M. Mints (1955) and others are known for the rate of constrained deposition in laminar flow. For example, for coal pulps, the laminar flow model was used in the works of A.I. Nazimko [4], O.D. Polulyakh [5] and many others.

However, laminar flow is practically not realized in dressing (enrichment) equipment. In practice, there is an unevenness of velocities both along the cross-section of the devices and in the transverse direction due to the joint fall of particles, the presence of walls of the device, circulation flows during power input, etc. It is obvious that during hydraulic classification and separation, they try to avoid turbulence of flows, since this hinders the separation processes. Because of this, the turbulent flow model does not correspond to real processes and is not acceptable.

It is physically reasonable to assume that weakly turbulent flows are realized in gravity dressers. The problem of determining the rate of constrained deposition in a weakly turbulent flow has no theoretical solution today and is characterized by the complexity of obtaining experimental and semi-experimental formulas, a narrow range of their application.

Problem and unsolved aspects. Starting with Darcy and Karman, a set of semi-empirical and empirical formulas for velocity under weakly turbulent flow regimes has been obtained [3, 6]. Complex models are known, for example, the laminar-turbulent flow zone is identified with a nonequilibrium phase transition, the mechanism of which is diffuse stratification [7], but the solutions obtained in this case are rather of academic interest. Despite a large number of studies, the problem today is that "there are no sufficiently substantiated formulas for the rate of constrained fall in the presence of heterogeneous particles in the pulp" [1].

In order to compare different formulas and choose the most acceptable one, you need to compare them with an experimental database. The problem is that this database should include a lot of experiments, since the variables should be - the size, the density of particles, and the characteristics of the suspension. However, in experiments for the same composition of particles and the density of the medium, the measured velocities differ numerically, which is related to the features of the method and equipment for measurement, particle shape, etc. [2]. Therefore, theoretical studies of various formulas are a promising means of analysis.

It is obvious that any formula for the rate of constrained deposition should give the value of the free fall velocity when moving to the appropriate characteristics of the medium. The free fall velocity is usually known from experiments, it is easier to measure and can serve as a reference point for the correct choice of the constrained fall formula. If this coincidence does not occur, it means that the latter has a narrow application, usually acceptable only for a certain concentration or collection of particles.

Purpose. The aim of the work is to propose an analytical method for comparing various formulas for the rate of constrained deposition, to show its application by the example of the well-known formulas of Ergan and Todes-Rosenbaum and to develop a new formula for the linear approximation of Ergan, which is suitable for practical calculations of the rate of constrained deposition in a wide range of suspension densities. The use of such a formula will improve the accuracy of velocity calculations, which is necessary for design and engineering work on the creation of hydraulic devices.

2. Methods - theoretical analysis and mathematical calculations.

Problem statement.

Many formulas of constrained deposition are based on the model of a granular suspension layer. The granular layer is a filling with a dense packing of particles, usually cubic or rhombohedral. Such a model is effective for describing liquid filtration and processes in a suspended (boiling) layer, for example, when drying raw materials in a fluidized suspended layer. The granular layer model is widely used in the calculation of chemical technology processes. For example, the formulas of N.I. Gelperin (1960), V.G. Einstein (1959), M.E. Aerov (1960), O.M. Planovsky (1967), O.M. Todes (1969), etc. are known. Among the many formulas for the deposition rate in a granular layer, the most well-known are the formulas of Ergan [8] and Todes-Rosenbaum [9], therefore, special attention should be paid to these formulas.

The Todes-Rosenbaum formula (1958) in the handbook of ore dressing [1] is recommended for calculating the rate of constrained deposition in dense pulps. It is given in the books of B.I. Brownstein and V.V. Shchegolev (1988), S.S. Zabrodsky (1964) and many others. This formula has the form:

$$\omega = \frac{\nu}{d} \cdot \frac{Ar \cdot \varepsilon^{4,75}}{18 + 0,59\sqrt{Ar \cdot \varepsilon^{4,75}}}$$

$$Ar = \frac{g \cdot \Delta \cdot d^3}{\nu^2}, \tag{1}$$

$$\Delta = \frac{\rho_{pat} - \rho_s}{\rho_s}$$

where ω – velocity, cm/s; *Ar* – Archimedes' criterion; *d* – equivalent particle diameter, cm; ρ_{pat} - the density of the precipitating particle, g/cm^3; ρ_s – density of the suspension, g/cm^3; ν- kinematic viscosity, cm^2/s; ε – the proportion of liquid or porosity of the suspension, units; g= 981 cm/s^2.

The Todes-Rosenbaum formula (1) is derived on the basis of the well-known Sabri Ergan equation (S. Ergun, 1952) [8, 9].

The Ergan equation characterizes the pressure loss of the ΔP per unit height of the granular layer h:

$$\frac{\Delta P}{h} = 150 \cdot \frac{(1-\varepsilon)^2}{\varepsilon^3} \cdot \frac{\omega \cdot \mu}{d^2} + 1,75 \cdot \frac{1-\varepsilon}{\varepsilon^3} \cdot \frac{\omega^2 \cdot \rho}{d}, \tag{2}$$

where the notation is the same as in formula (1), μ is the dynamic viscosity, $\mu = \nu \cdot \rho_s$. The constants 150 and 1.75 were obtained by Ergan during the processing of a number of experiments in which balls, cylinders, tablets, marble chips, sorted coke were used as particles [8].

Below we will consider the comparison of velocities according to formulas (1) and (2) for a wide range of suspension densities, and also perform an approximation to the conditions of free fall for them. This will clarify the range of their practical application.

The theoretical part.

The left side of the Ergan equation (2) is the hydraulic resistance. To determine it, starting with A. Darcy (1856) and T. Karman (1912), separate models for laminar and turbulent flows were developed. However, a mixed flow regime is implemented in the working area of hydraulic devices. For such an intermediate zone, when determining the hydraulic resistance, a phenomenological approach is used, which consists in formally combining the pressure loss in laminar and turbulent flows. Besides, the laws of linear filtration (Darcy, Kozeni) and nonlinear filtration (Darcy-Weisbach, Karman, etc.) are combined. The general view of the dependence of the specific hydraulic resistance on the velocity has the form:

$$\frac{\Delta P}{h} = A \cdot \omega + B \cdot \omega^2 . \tag{3}$$

The first summand reflects the influence of viscosity forces (laminar flow), the second – inertia forces (turbulent flow). The values of the coefficients *A, B* were studied experimentally repeatedly by different authors [3].

The same phenomenological approach was used by S. Ergan, while the Kozeni-Karman formula for the permeability coefficient was used to determine the coefficients *A* and *B.* Also, based on the hypothesis of J. Kozeni (1927) on the transition from a capillary to a cellular model, a transition was made from the parameters of the capillary channel to the parameters of particle size *d*, parameters ε, ρ_s, μ of suspension and the volume of the granular layer *V*. As a result, the formula (2) was obtained by Ergan.

To determine the left side of equation (2), we use the balance of forces equation:

$$\overrightarrow{F_{res}} = \overrightarrow{F_{gr}} + \overrightarrow{F_A},$$
$$F_{res} = \rho_{pat} \cdot V \cdot g - \rho_s \cdot V \cdot g, \qquad (4)$$
$$\frac{F_{res}}{S \cdot h} = \frac{\Delta P}{h} = \frac{1}{h} \cdot \left(\frac{F_{gr} - F_A}{S} \right)$$

where F_{res} - the force of hydraulic resistance, F_{gr} - the force of gravity, F_A is the Archimedean force, where forces are measured in newtons.

It should be noted, that it is the density of the mineral suspension that is included in F_A, and not the density of water, as a diluent agent.

Given that the volume of the layer $V = S \cdot h \cdot (1\text{-}\varepsilon)$, where *S* is the area of the layer, *h* is the height, (1- ε) - is the concentration of grains, we get:

$$\frac{\Delta P}{h} = \frac{1}{h} \left(\frac{\rho_{pat} \cdot g \cdot S \cdot h \cdot (1-\varepsilon)}{S} - \frac{\rho_s \cdot g \cdot S \cdot h \cdot (1-\varepsilon)}{S} \right) = (\rho_{pat} - \rho_s) \cdot g \cdot (1-\varepsilon). \qquad (5)$$

Let's equate the right-hand sides of equations (2) and (5) and take into account that $\mu = \nu \cdot \rho_s$, where μ is the dynamic viscosity, ν (cm^2/s) is the kinematic viscosity. After the transformations, we get:

$$\frac{1.75 \cdot \omega^2}{d} + 150 \cdot \omega \cdot \frac{(1-\varepsilon) \cdot \nu}{d^2} = g \cdot \varepsilon^3 \cdot \frac{\rho_{pat} - \rho_s}{\rho_s}. \qquad (6)$$

In equation (6), the first summand is the turbulent component, which reflects the inertia forces, the second - is the laminar component, the viscosity forces. This is the most common type of Ergan formula. If the particle diameter decreases, then both summands in the left part decrease, but the second summand decreases much faster. It means that the turbulent component grows faster. If the velocity decreases, then the second summand decreases more slowly, that is, the laminar component prevails during the flow It's in line with physics of the process.

We use the same notation for Δ as in formula (1):

$$\Delta = \frac{\rho_{pat} - \rho_s}{\rho_s}.$$

and we write equation (6) in the form:

$$1.75 \cdot \omega^2 \cdot d + 150 \cdot \omega \cdot (1-\varepsilon) \cdot \nu = g \cdot \varepsilon^3 \cdot \Delta \cdot d^2 . \quad (7)$$

The Ergan equation (7) is quadratic both with respect to velocity and with respect to size d. Below, it is for this equation that we will do calculations, that is, we will solve the quadratic equation with respect to velocity.

Unlike the Ergan equation, the Todes-Rosenbaum equation (1) is linear. In the original paper [9], as well as in [8], the derivation of equation (1) is described as follows. First, both parts of equation (6) are multiplied by $d^3 / (\rho \cdot \nu^2)$, then proceed to the quadratic equation with respect to variable $Re = \omega \cdot d / \nu$ with a free term in the form of Ar. Next, the method of linearization of the quadratic equation is used. It consists in discarding the *second linear term* in the equation, expressing ω from the resulting equation and substituting it as the second factor into the *first term* of the equation. After simplification, equation (1) is obtained.

In contrast to the original work [9], we applied the specified linearization method *directly* to equation (7), and considered two options - when discarding the first term of the equation (7) and the second. The second option turned out to be working, when we operate with the first, turbulent component (it is also proposed in [9]). The first option is not acceptable, because compared with equation (7), it gives about ten times less velocity.

In formula (7), we drop the second term, then we get for ω:

$$\omega = \sqrt{\frac{g \cdot \varepsilon^3 \cdot \Delta \cdot d^2}{1.75 d}} . \quad (8)$$

We use the definition of the concentration of solid particles in the suspension: $\beta = 1 - \varepsilon$ and introduce the notation:

$$B = g \cdot \varepsilon^3 \cdot \Delta \cdot d^2 .$$

Let's substitute expression (8) as the second term in the first term of equation (7):

$$1.75 \cdot \omega \cdot d \cdot \sqrt{\frac{B}{1.75 \cdot d}} + 150 \cdot \omega \cdot \beta \cdot \nu = B . \quad (9)$$

From here we express ω:

$$\omega = \frac{B}{150 \cdot \beta \cdot \nu + \sqrt{1.75 \cdot d \cdot B}} , \tag{10}$$

where $B = g \cdot \varepsilon^3 \cdot \Delta \cdot d^2$, $\beta = 1 - \varepsilon$, $\Delta = \frac{\rho_{pat} - \rho_s}{\rho_s}$.

The resulting formula (10), in comparison with the Todes-Rosenbaum equation (1), has a simpler, more convenient form for calculations. There is no reason to doubt the correctness of the derivation of formula (10), there is also no point in trying to bring it to the form (1).

Thus, theoretical analysis has shown that it is advisable to compare the calculation of the velocity according to the basic Ergan formula of the form (7) with the approximate formulas (1) of Todes-Rosenbaum and with the formula (10) obtained by us.

3. Results and discussion

Let's consider a simple deposition of a quartz particle with a grain size of d = 0.1 cm in a suspension consisting of a mixture of small particles of quartz sand with water. The particle density is equal to the weighted average density of the solid phase of the suspension. The suspension itself can have a different density $\rho_s = 1.25 \div 1.8$ g / cm^3.

Each density ρ_s corresponds to certain hydraulic characteristics of the medium. To determine them, we use the results of [10]. Hydraulic parameters of the medium: porosity ε, concentration β, viscosity ν are determined by the following equations [10]:

$$\varepsilon = \frac{\rho_m - \rho_s}{\rho_m - 1}, \quad \beta = 1 - \varepsilon, \quad \nu = \nu_0 \, exp \frac{2.5 \cdot \beta + 0.675 \cdot \beta^2}{1 - 0.609 \cdot \beta} \text{ (cm}^2\text{/s)}, \tag{11}$$

where ρ_m – weighted average density of suspension particles; ρ_s - suspension density; ε, β – porosity and concentration, respectively; $\nu_0 = 0{,}01$ cm^2/s – kinematic viscosity of water at 20° C.

Note that if particles of significantly different sizes are present in the pulp, ε should be determined, taking into account the yield of individual size classes, as a weighted average. However, since the mineral suspension in hydraulic apparatuses consists of finely ground raw materials, then at this stage we do not consider taking into account the size, in the value of ε.

It should also be noted that in the equations (1), (7), (10) the calculated values are:

$$\varepsilon = \frac{\rho_m - \rho_s}{\rho_m - 1}, \quad \Delta = \frac{\rho_{pat} - \rho_s}{\rho_s}.$$

In the first equation, ρ_m is the weighted average density of suspension particles. This is physically justified, since particles with different densities can be found in the suspension, and the porosity ε determines only the number of gaps between the particles.

In the second equation, ρ_{pat} is the density of a single particle which is deposited in this suspension. This is a consequence of using the balance of forces in the derivation of equation (7), where the forces act precisely on the precipitating particle.

Since we are considering the deposition of a quartz particle in a suspension from a mixture of quartz sand with water, then in this case $\rho_{part.} = \rho_m = 2.65$ g/cm^3.

Table 1 shows the indicators obtained by formulas (11). Then they are used for calculations for all three analyzed formulas (1), (7), (10).

Table 1 – Characteristics of water-sand suspension of different densities and parameters for calculating the rate of constrained deposition of quartz particles

ρ_s, g/cm^3	g, cm/s^2	d, cm	Δ	ε	β	ν, cm^2/s
1,15	981	0.1	1.304	0.9091	0.091	0.0128
1,25	981	0.1	1.120	0.8485	0.152	0.0154
1,3	981	0.1	1.038	0.8182	0.182	0.0171
1,4	981	0.1	0.893	0.7576	0.242	0.0213
1,5	981	0.1	0.767	0.6970	0.303	0.0273
1,6	981	0.1	0.656	0.6364	0.364	0.0361
1,7	981	0.1	0.559	0.5758	0.424	0.0492
1,75	981	0.1	0.514	0.5455	0.455	0.0584
1,8	981	0.1	0.472	0.5152	0.485	0.0699
1,9	981	0.1	0.395	0.4545	0.545	0.1041
1,002*	981	0.1	1.645	0.9992	0.001	0.01

* The bottom line is data for determining the rate of free deposition of quartz with a size of 1 mm in water.

Let's write the Ergan equation (7) as an ordinary quadratic equation:

$$a\,\omega^2 + в\,\omega + c = 0, \quad a = 1.75;\ в = 150\cdot(1-\varepsilon)\cdot\nu/d;\ \ c = -\,g\cdot\varepsilon^3\cdot\Delta\cdot d.$$

The discriminant D of this equation is always positive, so it has two radicals ω_1, ω_2:

$$D = b^2 - 4\cdot a\cdot c, \quad D>0 \rightarrow\ \omega_1 = \frac{-b+\sqrt{D}}{2a}, \quad \omega_2 = \frac{-b-\sqrt{D}}{2a}.$$

Since the second radical is negative, we calculate only the first one - ω_1. The calculations of the Ergan velocity were performed using the data from Table 1 in the Ms. Excel program and are shown in Table 2.

Table 2 – The rate of constrained deposition according to the Ergan formula for a quartz particle with a size of 1 mm in a water-sand mixture of different densities

Re	ρ_s, g/cm^3	*a*	*b*	*c*	*D*	$\sqrt{D}$	ω, cm/s
54.2	1.15	1.75	1.745	-96.14	675.99	26.0	6.93
34.1	1.25	1.75	3.509	-67.11	482.12	22.0	5.27
26.1	1.3	1.75	4.662	-55.80	412.3	20.3	4.47
13.8	**1.4**	1.75	7.757	-38.08	326.7	18.1	2.95
6.08	**1.5**	1.75	12.418	-25.46	332.5	18.2	1.66
2.19	**1.6**	1.75	19.664	-16.59	502.8	22.4	0.79
0.67	1.7	1.75	31.328	-10.46	1054.7	32.5	0.33
0.35	1.75	1.75	39.797	-8.19	1641.1	40.5	0.20
0.18	1.8	1.75	50.871	-6.33	2632.2	51.3	0.12
0.04	1.9	1.75	85.164	-3.64	7278.4	85.3	0.04
95.7*	**1.002***	1.75	0.012	-160.95	1126.7	33.6	**9.59**

* The bottom line is the calculation according to the formula (7) for free deposition rate of quartz with a size of 1 mm in water.

Table 2 shows the Reynolds number on the left *Re =ωd/ν.* As we can see, with an increase in the density of the suspension, *Re decreases* non-linearly. This means that the influence of viscosity and friction forces *increases*, which is typical for laminar flows. However, the physics of the process says that with an increase in the concentration of particles, the inertia (braking) forces (inherent in turbulent flows) should increase to a greater extent. It follows that for constrained deposition, it is necessary to operate with caution with the Reynolds number.

The velocity calculations according to the Todes-Rosenbaum formula (1) were performed using the indicators of Table 1, Ms. Excel program and are shown in Tab.3.

Table 3 – The rate of constrained deposition according to the Todes-Rosenbaum formula for a quartz particle with a size of 1 mm in a suspension of different densities

ρ_c, g/cm^3	*Ar*	$\varepsilon^{4,75}$	*k*	ω, cm/s	Difference (ω - velocity by Ergan), cm/s
1.15	7815.5	0.636	0.13	10.671	3.74
1.25	4608.9	0.458	0.15	7.228	1.96
1.3	3485.7	0.386	0.17	5.797	1.33
1.4	1925.0	0.267	0.21	3.499	0.55
1.5	1007.6	0.180	0.27	1.910	0.25
1.6	495.4	0.117	0.36	0.928	0.14
1.7	226.2	0.073	0.49	0.397	0.07
1.75	148.1	0.056	0.58	0.246	0.04
1.8	94.7	0.043	0.70	0.148	0.02
1.9	35.7	0.024	1.04	0.047	0.00
1.002*	16069.0	0.996	0.10	**17.31**	7.73

* The bottom line is the calculation according to the formula (1) for free deposition rate of quartz with a size of 1 mm in water.

The calculation of the velocity according to the approximate formula (10) obtained by us for the indicators of Table 1 is given in Table 4.

Table 4 - The rate of constrained deposition according to formula (10) for a quartz particle with a size of 1 mm in a water-sand mixture of different densities

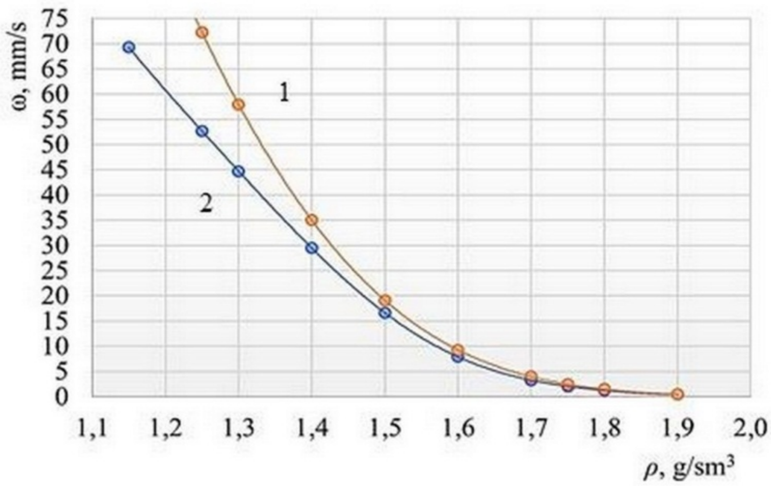
ρ_s, g/cm^3	B	ω, cm/s	Difference (ω - velocity by Ergan), cm/s
1.15	9.61	6.53	-0.40
1.25	6.71	4.68	-0.59
1.3	5.58	3.84	-0.63
1.4	3.81	2.39	-0.56
1.5	2.55	1.33	-0.33
1.6	1.66	0.66	-0.13
1.7	1.05	0.29	-0.03
1.75	0.82	0.19	-0.02
1.8	0.63	0.12	-0.01
1.9	0.36	0.04	-0.00
1.002*	16.10	**9.58**	-0.00

* The bottom line is the calculation according to the formula (10) for free deposition rate of quartz with a size of 1 mm in water.

Tables 3, 4 show that formula (10) gives a better approximation to the original Ergan formula, than the Todes-Rosenbaum formula.

An important criterion for choosing a computational equation is the condition that it should give at least one value known from experiments. For this value, we take the rate of free deposition of a quartz particle with a size of 1 mm in water. This velocity, according to various sources, is 95-100 mm/s. Thus, according to Richards' experiments [2], the free fall velocity of quartz grains of 1.04 mm in water is 95 mm/s, according to [1] this velocity is 10 cm/s.

Tables 2-4 show that the Todes-Rosenbaum formula does not allow obtaining the specified experimental value. This gives reason to doubt the advisability of its use. Figure 1 allows to clarify the range of applicability of this formula.



1 - according to the formula (1) Todes-Rosenbaum; 2 – according to the formula (7) Ergan

Figure 1 – Dependence of the rate of constrained deposition of a quartz particle with a size of 1 mm on the density of the suspension

Figure 1 shows that the best match of the Todes-Rosenbaum formula with the original Ergan formula occurs for dense suspensions, at $\rho_s > 1.65$ g/cm^3. Such dense suspensions are not suitable and are not used for mineral pulp dressing processes. For dilute suspensions, the Todes-Rosenbaum formula gives inflated values. So, at $\rho_s = 1.2$ g/cm^3, the discrepancy is up to 30%.

Thus, to estimate the rate of constrained deposition, it is advisable to use either the original Ergan formula (7) directly, or the simplified formula (10) obtained by us.

4. Conclusions

During hydraulic classification and separation, a mixed, laminar-turbulent flow mode is observed in real pulps. Many semi-experimental and experimental formulas are known to calculate the rate of constrained deposition in this mode. New method has been developed for comparing different formulas with each other in a wide range of suspension densities.

This method differs in that it uses an analytical calculation of the hydraulic characteristics of the suspension depending on the density according to formulas (11), the presence of dissimilar particles in the pulp is taken into account and only one control point corresponding to the conditions of free deposition of the particle in water is used for comparison with the experiment.

This method allows to set the limits of the application of formulas depending on the density of the suspension. The choice of a more precise formula is necessary for the design development, determination of optimal modes and monitoring of performance indicators of hydraulic devices for the classification and separation of mineral suspensions and finely ground composite raw materials.

Today, the most physically justified is the quadratic formula of Ergan. The comparison of the Ergan formula and the well-known Todes-Rosenbaum formula obtained with its linear approximation was performed. It was established that the Todes-Rosenbaum formula is actually not acceptable for dilute suspensions that are used for mineral pulps during dressing.

A new linear formula for calculating the rate of constrained deposition is proposed, which gives a high approximation to the Ergan formula over the entire range of suspension densities. In contrast to the Todes-Rosenbaum equation, the equation obtained by us, when approaching the conditions of free deposition of quartz in water, gives a good match with experimental data.

For practical calculations, it is recommended to use formulas (10) and (11), which give a system of simple equations for determining the velocity of the constrained fall of a particle of arbitrary size and density in a wide range of densities of suspensions, for example, mineral pulps or aqueous suspensions of finely ground materials.

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ШВИДКІСТЬ СТИСНЕННОГО ОСАДЖЕННЯ ЧАСТИНОК В ШИРОКОМУ ДІАПАЗОНІ ГУСТИНИ СУСПЕНЗІЇ ПРИ ЛАМІНАРНО-ТУРБУЛЕНТНОМУ РЕЖИМІ

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Анотація. Швидкість стисненого падіння мінеральних частинок в суспензіях різної густини необхідна для розрахунку конструкції та режимів роботи гравітаційного збагачувального обладнання. При гідравлічній класифікації та сепарації у реальних пульпах спостерігається змішаний, ламінарно-турбулентний режим течії. Для такого режиму теоретичні формули швидкості відсутні, а більшість відомих напівекспериментальних та експериментальних формул мають обмежене застосування. У цій статті запропоновано новий метод порівняння різних формул між собою в широкому діапазоні щільності суспензій. В методі використовується аналітичний розрахунок гідравлічних характеристик середовища - порозності, концентрації та в'язкості. Новим є те, що всі ці характеристики залежать тільки від одного показника - густини суспензії, яка легко визначається на практиці методом зважування проби пульпи. В цих розрахунках використовується середньозважена щільність різнорідних частинок суспензії. Метод відрізняється наближенням аналізованих розрахункових формул до умов вільного падіння з метою отримання лише однієї контрольної точки для порівняння її з відомими експериментальними даними. Цей метод дозволяє встановити межі застосування формул в залежності від густини суспензії. Вибір більш точної формули необхідний для проектування гідравлічних пристроїв для класифікації та розділення мінеральних суспензій і тонко подрібненої композиційної сировини. Показано застосування даного методу для найпоширеніших формул Ергана та Тодеса-Розенбаума. Встановлено, що остання формула фактично не придатна для розбавлених суспензій з густиною нижче 1,65 г/см3. Запропоновано нове лінійне рівняння розрахунку швидкості стисненого осадження, яке забезпечує високу точність в широкому діапазоні густини суспензії. Отримане рівняння має простий вигляд і в сукупності з аналітичним розрахунком характеристик середовища утворює систему простих лінійних рівнянь для розрахунку швидкості стисненого осадження частинок будь-якої крупності та щільності в мінеральних пульпах різної густини. Розрахунок швидкості стисненого осадження і спливання частинок покладено в основу проектування гідравлічних класифікаторів і сепараторів для збагачення корисних копалин. Такі розрахунки необхідні для визначення оптимальних режимів гідравлічних пристроїв і контролю показників під час їх роботи.

Ключові слова: мінеральна суспензія, щільність, швидкість стисненого осадження.